Shun advice at any price. That's what I call good advice.

Piet Hein

Final Exam: July 30 - 9:00 AM - 12:00 PM in ENW 117


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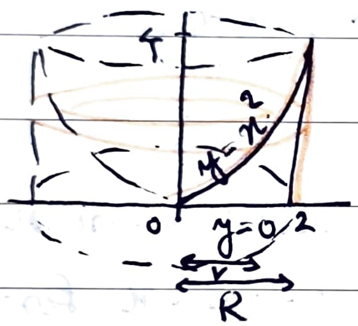
ellipses



revolve the region below $y=x^2$ & above $y=0$ $(0 \leq x \leq 2)$ above the y-axis.

$R = 2 - 0 = 2$

$r = x - 0 = x = \sqrt{y}$ (because $y = x^2 \Rightarrow x = \sqrt{y}$)

$A(y) = \pi(R^2 - r^2) \Rightarrow \pi(2^2 - (\sqrt{y})^2) \Rightarrow \pi(4-y)$

$V = \int_0^4 A(y)\,dy = \int_0^4 \pi(4-y)\,dy = \pi\left(4y - \frac{y^2}{2}\right)\Big|_0^4 \Rightarrow$

$\pi\left(4(4) - \frac{4^2}{2}\right) - \cancel{\pi\left(4(0) - \frac{0^2}{2}\right)} \Rightarrow \pi(16-8) = \boxed{8\pi}$

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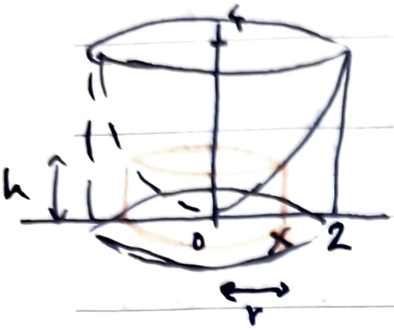
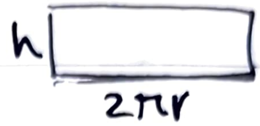
Another way to solve it:



$A = 2\pi r h$



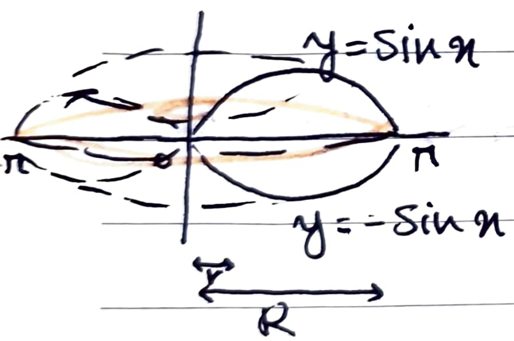
$r = x - 0 = \boxed{x}$

$h = y - 0 = y = \boxed{x^2}$

Therefore: $A = 2\pi x(x^2) \Rightarrow A = 2\pi x^3$

$$V = \int_0^2 A(x)\,dx \Rightarrow \int_0^2 2\pi x^3\,dx \Rightarrow 2\pi\left(\frac{x^4}{4}\right) = \frac{\pi}{2}x^4\Big|_0^2$$

$$\Rightarrow \frac{\pi}{2}(2)^4 - \cancel{\frac{\pi}{2}(0)^4} \Rightarrow \frac{16\pi}{2} = \boxed{8\pi}$$



Region below $y = \sin x$ & above $y = -\sin x$ for $0 \leq x \leq \pi$ revolve about y-axis.

$r = \arcsin(y)$

$R = \pi - \arcsin(y)$

$$A(y) = \pi(R^2 - r^2) \Rightarrow$$

$$\pi((\pi - \arcsin(y))^2 - \arcsin^2(y))$$

$$\Rightarrow \pi(\pi^2 - 2\pi\arcsin(y) + \cancel{\arcsin^2(y)} - \cancel{\arcsin^2(y)}$$

Becomes much difficult! so instead:

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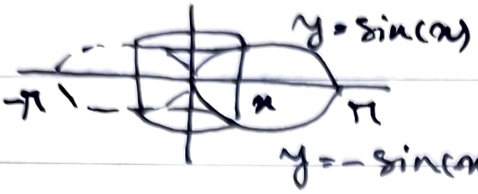


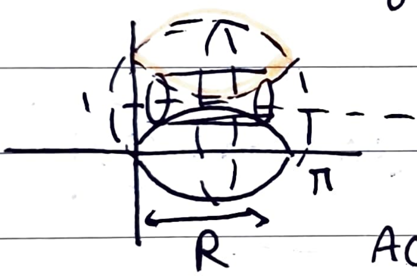
$h = \sin(x) - (-\sin(x)) \Rightarrow 2\sin(x)$

$r = x - 0 = x$

$A(x) = (2\pi)\, 2x \sin x \Rightarrow 4\pi x \sin(x)$

$V = \int_0^{\pi} 4\pi x \sin(x)\, dx$

* How about revolving above $y = 1$?



$R = \pi - \arcsin(y) - \arcsin(y)$

$r = 1 - y$

$A(y) = \pi(R^2 - r^2) =$

$\pi((\pi - 2\arcsin(y))^2 - (1-y)^2) \ldots$

so instead:

$R = 1 - (-\sin(x)) = 1 + \sin(x)$

$r = 1 - \sin(x)$

$A(x) = \pi((1+\sin(x))^2 - (1-\sin(x))^2) = \pi(2.2\sin(x))$

$\Rightarrow 4\pi \sin(x)$

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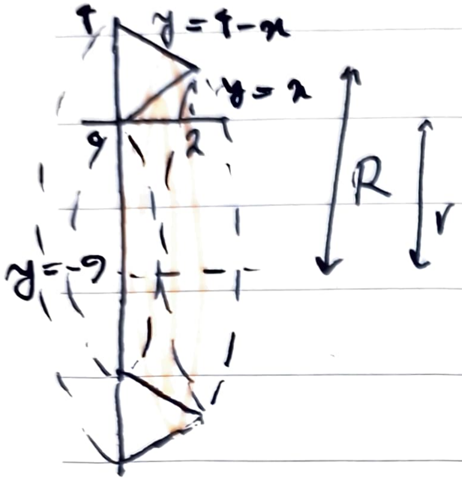
Region above $y=x$, below $y=4-x$

$0 \leqslant x \leqslant 2$, $y=-9$



$R=(4-x)-(-9) \Rightarrow \boxed{13-x}$

$r=x-(-9) \Rightarrow \boxed{x+9}$

$A(x)=\pi(R^2-r^2) \Rightarrow$

$\pi((13-x)^2-(9+x)^2) \Rightarrow$

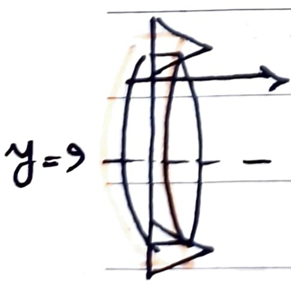
$\pi((169-26x+x^2)-(81+18x+x^2)) \Rightarrow$

$\pi(88-44x)$

$$V=\int_0^2 \pi(88-44x)\,dx \Rightarrow \pi\left(88x-44\frac{x^2}{2}\right)\Big|_0^2 \Rightarrow$$

$$\pi(176-88)-\cancel{\pi(0)} \Rightarrow \boxed{88\pi}$$

Now doing it with shells:



$R=x-(-9)$

$h=x-0$

$A=2\pi rh=2\pi x(x+9)$ Not enough!

so we need a bigger shell:

$r=(4-x)-(-9) \Rightarrow 13-x \Rightarrow 9+y$

$h=4-y$

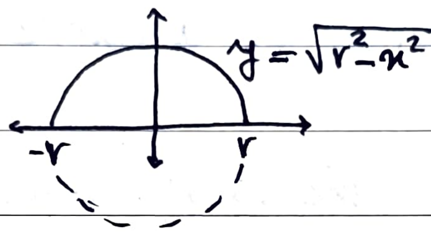
$r=x+9=13-y$

$h=x=y$

$V = \frac{4}{3}\pi r^3$

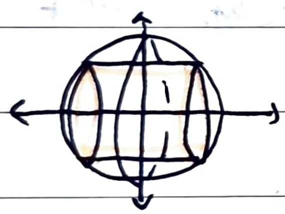
How is the sphere a solid of revolution?

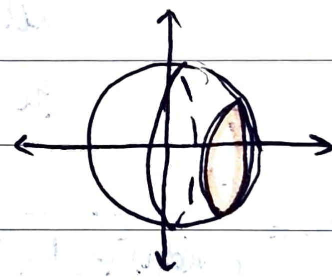
Revolve the region between $y = 0$ for $-r \leq x \leq r$ about the x-axis.



with shells:

with washer/disks:





→ with disks:

$R = y - 0 = y = \sqrt{r^2 - x^2}$

$A(x) = \pi R^2 = \pi\left(\sqrt{r^2 - x^2}\right)^2 = \pi(r^2 - x^2)$

$$V = \int_{-r}^{r} A(x)\,dx = \int_{-r}^{r} \pi(r^2 - x^2)\,dx = \pi\left(r^2 x - \frac{x^3}{3}\right)\Bigg|_{-r}^{r} \Rightarrow$$

$$\pi\left(r^2(r) - \frac{r^3}{3}\right) - \pi\left(r^2(-r) - \frac{-r^3}{3}\right) = \pi\left(\frac{2}{3}r^3\right) - \pi\left(-\frac{2}{3}r^3\right)$$

$$\Rightarrow \boxed{\frac{4}{3}\pi r^3}$$

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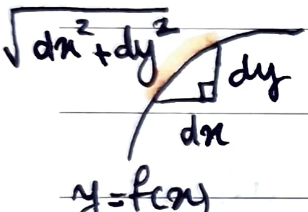
The surface area of a sphere $\Rightarrow A = 4\pi r^2$
but why?

Computing arc-length:
(length of a piece of a curve).



The length of the curve from $x=a$ to $x=b$ is adding up all these $\sqrt{(dx)^2+(dy)^2}$ along the way.

$$\text{Ave-length} = \int_a^b \sqrt{(dx)^2+(dy)^2} = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_a^b \sqrt{1+[f(x)]^2}\,dx$$

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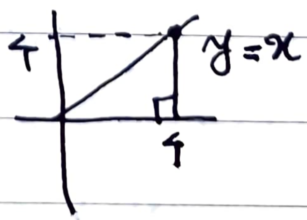
what is the length of $y=x$ for $0 \leq x \leq 4$?

$$\frac{dy}{dx} = \frac{d}{dx} x = 1$$

$$\text{Ave-length} = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^4 \sqrt{2}\,dx = \sqrt{2}(x)\Big|_0^4 =$$

$$4\sqrt{2} - \cancel{0\sqrt{2}} \Rightarrow 4\sqrt{2}$$

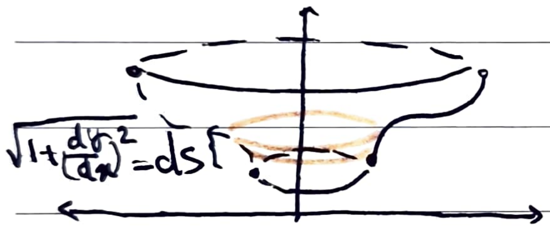


$y = 12x^2 \quad -4 \leqslant x \leqslant 7 \qquad \frac{dy}{dx} = 24x$

$$\text{arc-length} = \int_{-4}^{7} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_{-4}^{7} \sqrt{1+576x^2}\, dx$$

take 1120 H to solve this! :)

Areas of surfaces of revolution:

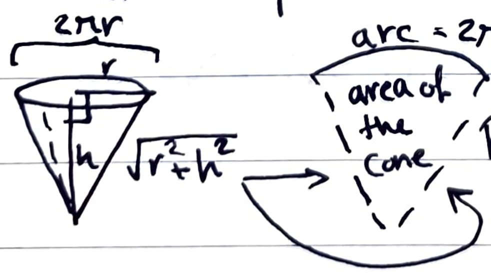


surface of revolution

$$\text{Area} = 2\pi r\left(\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right)$$

$$\text{Surface Area} = \int_{0}^{b} 2\pi r \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

Area of an open cone:



$R = \sqrt{r^2+h^2}$

Area of the cone = $\dfrac{\text{area of wedge}}{\text{area of the big circle}}$

$\pi R^2 = \pi\left(\sqrt{r^2+h^2}\right)$

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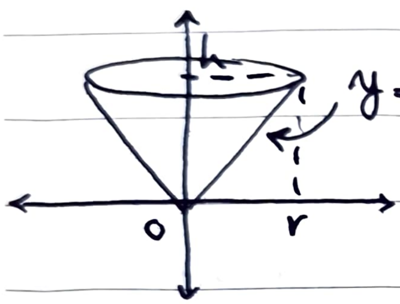
which can also be written as: $\dfrac{\text{arc of wedge}}{\text{circumference of the big circle}}$

therefore $= \dfrac{2\pi r}{2\pi R}(\pi R^2) = \pi r\sqrt{r^2+h^2}$

so area of an open cone $= \pi r\sqrt{r^2+h^2}$



$y = \frac{h}{r}x$ so $\frac{dy}{dx} = \frac{h}{r}$

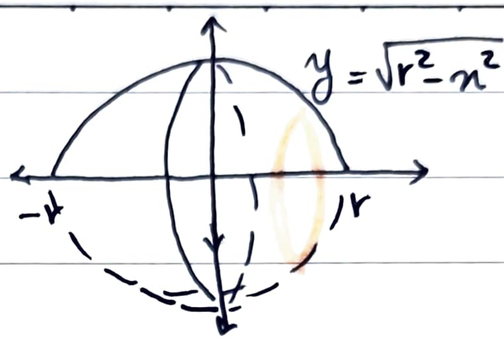
Plug into SA equation:

$$A = \int_0^r 2\pi x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \Rightarrow 2\pi\int_0^r x\sqrt{1+\left(\frac{h}{r}\right)^2}\,dx \Rightarrow$$

$$2\pi\sqrt{1+\left(\frac{h}{r}\right)^2}\int_0^r x\,dx \Rightarrow 2\pi\sqrt{1+\left(\frac{h}{r}\right)^2}\left(\frac{x^2}{2}\right)\Bigg|_0^r \Rightarrow$$

$$\pi\sqrt{1+\left(\frac{h}{r}\right)^2}(r^2) - \cancel{\pi\sqrt{1+\left(\frac{h}{r}\right)^2}(0)^2} \Rightarrow \boxed{\pi r\sqrt{r^2+h^2}}\ \checkmark$$

★ Now the sphere!



$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{r^2 - x^2} \Rightarrow$$

$$\frac{d}{dx}(r^2 - x^2)^{1/2} \Rightarrow \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}\left(\frac{d}{dx}(r^2 - x^2)\right) = \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}(-2x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{r^2 - x^2}}$$

$$A = \int_{-r}^{r} 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \Rightarrow 2\pi \int_{-r}^{r} \sqrt{r^2 - x^2}\left(\sqrt{1 + \left(\frac{-x}{\sqrt{r^2 - x^2}}\right)^2}\right) dx$$

$$\Rightarrow 2\pi \int_{-r}^{r} \sqrt{r^2 - x^2}\left(\sqrt{1 + \frac{x^2}{r^2 - x^2}}\right) dx \Rightarrow 2\pi \int_{-r}^{r} \sqrt{r^2 - x^2 + x^2}\, dx \Rightarrow$$

$$2\pi \int_{-r}^{r} r\, dx \Rightarrow 2\pi r x \Big|_{-r}^{r} \Rightarrow 2\pi r(r) - 2\pi r(-r) \Rightarrow$$

$$2\pi r^2 + 2\pi r^2 = \boxed{4\pi r^2} \checkmark$$